ACOUSTIC INTERACTION OF TWO CYLINDERS

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In this paper we investigate several properties of acoustic waves propogating along a circular cylinder oscillating near a screen or a free surface. In determining the values of the parameters a sharp change in the intensity of the acoustic field is observed due to an acoustic resonance between the oscillating cylinder and the intrinisic oscillations of the gas in the relative region.

We consider the problem of the acoustic interaction of two circular cylinders of radius R oscillating according to a given harmonic law with small amplitude. We assume the medium to be ideal and compressible and the motion of the medium to be plane-parallel and potential. We take a rectangular coordinate system for each cylinder with coordinates x_n, y_n ($n=1, 2$) measured from the center of the cylinder axis. The axes y_1, y_2 are taken along the line joining the origin of the coordinates with

$$x_1 = x_2, \quad y_1 = y_2 - H \tag{1}$$

where H is the distance between the cylinder axes (Fig. 1).

Below, as the basic coordinate system we choose x_1, y_1, denoting these coordinates as x, y.

We assume that the velocity potential φ (x, y, t) can be represented in the form

$$\varphi(x, y, t) = Ra\Phi(x, y)e^{i\omega t} \tag{2}$$

where Φ (x, y) is the dimensionless amplitude function of the velocity potential, a is the velocity of sound, and ω is the angular frequency of the oscillating cylinders. Then, within the limits of the assumptions made here, the function Φ satisifies the Helmholtz equation

$$\Phi_{xx} + \Phi_{yy} + k^2\Phi = 0 \qquad (k = \omega R/a) \tag{3}$$

and the boundary conditions

$$\partial\Phi/\partial r_n = F_n(\theta_n) \quad \text{for} \quad r_n = 1 \quad (n = 1, 2) \tag{4}$$

$$\lim_{r_n\to\infty} r_n^{1/2}\Phi = 0, \qquad \lim_{r_n\to\infty} r_n^{1/2}\left(\frac{\partial\Phi}{\partial r_n} - ik\Phi\right) = 0 \tag{5}$$

Here r_n, θ_n are the dimensionless polar coordinates related to x_n, y_n by the equations

$$x_n = Rr_n\cos\theta_n, \quad y_n = Rr_n\sin\theta_n \quad (n = 1, 2) \tag{6}$$

and $F_n(\theta_n)$ are the dimensionless amplitude functions of the normal components of the velocity for points along the circumference of the n-th cylinder:

$$\partial\varepsilon_n / \partial t = aF_n(\theta_n)e^{i\omega t} \tag{7}$$

where ε_1, ε_2 are the normal components of the displacement vector of the points along the circumference of the first and second cylinder, respectively.

The solution of Eqs. (3)-(5) will be found by the interference method [1]. We represent Φ in the form

$$\Phi(x, y) = \sum_{n=1}^{2} [\Phi_n(r_n, \theta_n) + \Psi_n(r_n, \theta_n)] \tag{8}$$

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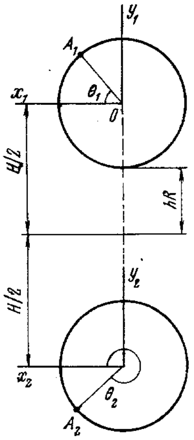

Fig. 1

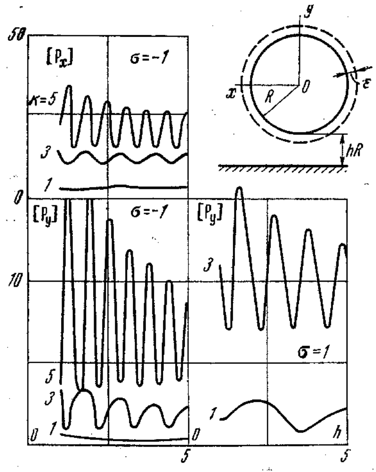

Fig. 2

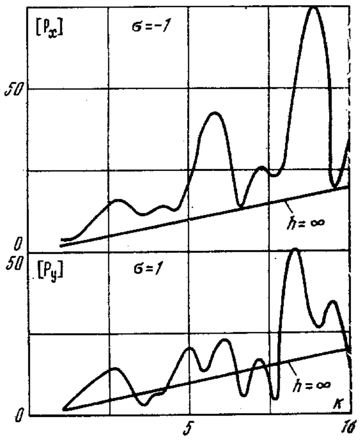

Fig. 3

Here Φ_n is the amplitude function of the velocity potential of the streamline of a single cylinder oscillating according to the given law in Eq. (7). The function Ψ_n also corresponds to the flow near a single cylinder oscillating according to some other law whose introduction permits one to take account of the mutual influence (interference) of the cylinders. This unknown law for the oscillation of cylinders is determined from the nonflow condition in Eq. (4).

The solution of the problem for the function Φ_n satisfying Eq. (3) and the conditions in Eqs. (4) and (5) has the form

$$\Phi_n(r_n, \theta_n) = \frac{2}{k} \sum_{m=0}^{\infty} H_m^{(2)}(kr_n) \frac{a_m^{(n)} \cos m\theta_n + b_m^{(n)} \sin m\theta_n}{H_{m-1}^{(2)}(k) - H_{m+1}^{(2)}(k)} \tag{9}$$

Here $a_m^{(n)}$, $b_m^{(n)}$ are the Fourier coefficients for the function $F_n(\theta_n)$, and $H_m^{(2)}$ is the Hankel function of second order.

The function $\Psi_n(r_n, \theta_n)$ is determined from the same Eq. (9), and instead of the known coefficients $a_m^{(n)}$, $b_m^{(n)}$, it is necessary to substitute the desired coefficients $c_m^{(n)}$, $d_m^{(n)}$. The equations used for the determination of these coefficients are obtained from the nonflow condition in Eq. (4) being substituted into Eq. (8):

$$\frac{\partial \Phi_n}{\partial r_j} + \sum_{p=1}^{2} \frac{\partial \Psi_p}{\partial r_j} = F_j(\theta_j) \quad \text{for } r_j = 1 \ (j, n = 1, 2;\ n \neq j) \tag{10}$$

The solution of the system of equations will be found for the case of oscillations of the cylinder near the screen and a free surface.

For an oscillating cylinder near a screen the effect of the screen can be taken into account as a second cylinder indentified to the mirror reflection of the first cylinder with respect to the screen. Then, at the corresponding points A_1 and A_2 the condition

$$F_1(\theta_1) = F_2(\theta_2) \quad \text{for } \theta_2 = 2\pi - \theta_1 \tag{11}$$

or

$$\sum_{m=0}^{\infty} (a_m^{(1)} \cos m\theta_1 + b_m^{(1)} \sin m\theta_1) = \sum_{m=0}^{\infty} (a_m^{(2)} \cos m\theta_1 - b_m^{(2)} \sin m\theta_1)$$

will be satisfied.

From this it follows that the oscillation law of the second cylinder is related to the oscillations of the first cylinder by the equations

$$a_m^{(1)} = a_m^{(2)}, \qquad b_m^{(1)} = -b_m^{(2)} \tag{12}$$

For the case of an oscillating cylinder near a free surface which is assumed to be plane, the effect of the surface can also be described by substituting a second cylinder located symmetrically relative to this surface. It is then possible to show that at the points A_1 and A_2 the condition

$$F_1(\theta_1) = -F_2(\theta_2) \text{ for } \theta_2 = 2\pi - \theta_1 \tag{13}$$

must be satisfied, which leads to the equations

$$a_m^{(1)} = -a_m^{(2)}, \qquad b_m^{(1)} = b_m^{(2)} \tag{14}$$

Equations (12) and (14) permit Eq. (10) to be reduced to a single equation of the type

$$\sum_{n=0}^{\infty} \{c_n^{(1)}[M_n(\theta_1) + \sigma\cos n\theta_1] + d_n^{(1)}[N_n(\theta_1) + \sigma\sin n\theta_1]\} = -\sum_{n=0}^{\infty}[a_n^{(1)}M_n(\theta_1) + b_n^{(1)}N_n(\theta_1)] \tag{15}$$

in the case considered here.

Here $\sigma = -1$ for the screen and $\sigma = 1$ for the free surface:

$$M_n(\theta_1) = -\frac{k/z}{H_{n-1}^{(2)}(k) - H_{n+1}^{(2)}(k)}\left\{(1 + h_1\sin\theta_1)\cos n\theta_2[H_{n-1}^{(2)}(z) - H_{n+1}^{(2)}(z)] + \frac{2nh_1}{z}\cos\theta_1\sin n\theta_2 H_n^{(2)}(z)\right\}$$
$$N_n(\theta_1) = \frac{k/z}{H_{n-1}^{(2)}(k) - H_{n+1}^{(2)}(k)}\left\{(1 + h_1\sin\theta_1)\sin n\theta_2[H_{n-1}^{(2)}(z) - H_{n+1}^{(2)}(z)] - \frac{2nh_1}{z}\cos\theta_1\cos n\theta_2 H_n^{(2)}(z)\right\} \tag{16}$$

where

$$h_1 = 2(1 + h) = H/R, \qquad z = k\sqrt{1 + 2h_1\sin\theta_1 + h_1^2} \tag{17}$$

and the variable is related to θ_2, θ_1 by the equations

$$\cos\theta_2 = kz^{-1}\cos\theta_1, \quad \sin\theta_2 = kz^{-1}(h_1 + \sin\theta_1) \tag{18}$$

It is interesting to calculate the pressure p in the acoustic field along the oscillating cylinders. Neglecting quadratic perturbations in the velocity, we have from the Cauchy–Lagrange integral:

$$p - p_\infty = -\rho(\partial\varphi/\partial t) \qquad (p_\infty = \text{const}) \tag{19}$$

where ρ is the density of the medium.

Taking into account Eqs. (2) and (9), Eq. (19) assumes the form

$$p - p_\infty = {}^1\!/_2\rho a^2 C_p, \qquad C_p = -2ik\Phi(x, y)e^{i\omega t} \tag{20}$$

$$\Phi(x, y) = \frac{2}{k}\sum_{n=0}^{\infty}[H_{n-1}^{(2)}(k) - H_{n+1}^{(2)}(k)]^{-1}\{(a_n^{(1)} + c_n^{(1)})[H_n^{(2)}(kr_1)\cos n\theta_1 -$$
$$- \sigma H_n^{(2)}(kr_2)\cos n\theta_2] + (b_n^{(1)} + d_n^{(1)})[H_n^{(2)}(kr_1)\sin n\theta_1 + \sigma H_n^{(2)}(kr_2)\sin n\theta_2]\} \tag{21}$$

The pressure along the cylinders in the directions of the x and y axes is written, respectively, in the form

$$p - p_\infty = {}^1\!/_2\rho a^2 P_x|x|^{-1/2}e^{i\omega t} \quad (|x| \gg R,\ |y| \sim R)$$
$$p - p_\infty = {}^1\!/_2\rho a^2 P_y|y|^{-1/2}e^{i\omega t} \quad (|x| \sim R,\ |y| \gg R) \tag{22}$$

The moduli of the amplitude functions of the pressure $|P_x|$, $|P_y|$ is of practical interest. Using the asymptotic representation for the Hankel functions we have

$$|P_x| = 4\sqrt{\frac{1}{\pi k}}(1 - \sigma)\left|\sum_{n=0}^{\infty}\frac{a_n^{(1)} + c_n^{(1)}}{H_{n-1}^{(2)}(k) - H_{n+1}^{(2)}(k)}\right| \tag{23}$$

$$|P_y| = 4\sqrt{\frac{2}{\pi k}}\left|\sum_{j=0}^{\infty}(-1)^j\left[\frac{b_{2j+1}^{(1)} + d_{2j+1}^{(1)}}{H_{2j}^{(2)}(k) - H_{2j+2}^{(2)}(k)}(1 + \sigma) + \frac{a_{2j}^{(1)} + c_{2j}^{(1)}}{H_{2j-1}^{(2)}(k) - H_{2j+1}^{(2)}}(1 - \sigma)\right]\right| \tag{24}$$

In the special case of the oscillations of a single cylinder in an unbounded medium $\sigma = 0$, $c_n^{(1)} = d_n^{(1)} = 0$.

To carry out the calculations, the law of oscillations of the cylinder considered here is taken in the form

$$\varepsilon_1(\theta_1, t) = Re f(\theta_1) e^{i\omega t} \quad (\varepsilon = \text{const}) \tag{25}$$

Then, the normal component of the velocity of the cylinder is equal to $\partial \varepsilon_1/\partial t$, and the function $F_1(\theta) = ik\varepsilon f(\theta_1)$. As an example, we choose the form of oscillations of the cylinder

$$f(\theta_1) = \cos m\theta_1 \quad (a_n^{(1)} = i\delta_{nm}k\varepsilon,\ b_n^{(1)} = 0,\ m = 0, 1, 2, 3)$$

$$f(\theta_1) = \sin m\theta_1 \quad (a_n^{(1)} = 0,\ b_n^{(1)} = i\delta_{nm}k\varepsilon,\ m = 1, 2, 3) \tag{26}$$

where δ_{nm} is the Kronecker delta.

The functions Ψ_1, Ψ_2 are approximated by a finite number of terms with complex coefficients $c_n^{(1)}$, $d_n^{(1)}$ (n=0, 1, ..., N) which are determined by the collation method by satisfying Eq. (15) at 2N+1 equally spaced points. The calculation is carried out for N=10 (basis) and N=7 (control). The results of the calculations in both cases practically coincide (over the range of variation of the parameters h and K considered). In addition to the coefficients $c_n^{(1)}$, $d_n^{(1)}$, the values of $|P_x|$, $|P_y|$ characterizing the pressure in the acoustic field along the cylinders are calculated. Several results of calculations (for oscillations of the form $f=1$) are shown in Figs. 2 and 3. The screen corresponds to $\sigma=-1$, and the free surface to $-\sigma=1$. The case $h=\infty$ corresponds to oscillations of a single cylinder in an unbounded medium. A characteristic property of all calculations is the sharp increase of $|P_x|$, $|P_y|$ for specific values of the parameters k and h changing periodically with the quantity $kh \approx \pi$. It is interesting to note that such a period in the change is characteristic for the zeroes of the Bessel function. This property of the solution makes it possible to postulate that the periodic growth of the pressure amplitude far from the system of two oscillating cylinders is due to an acoustic resonance at the natural frequency of the unbounded medium whose form is

$$\Phi = I_n(kr)\begin{Bmatrix}\sin n\theta\\ \cos n\theta\end{Bmatrix} \qquad (n = 0, 1, \ldots) \tag{27}$$

where k is an arbitrary positive number, and r, θ give a polar coordinate system with its origin at the point $x_1=0$, $y_1=-H/2$.

In particular, for an oscillation of a cylinder close to a screen ($\theta = 0, \pi$), a resonance with natural frequencies of the unbounded liquid can arise for an oscillation form of $\Phi = I_0(kr)$ or $\Phi = I_2(kr)\cos 2\theta$ which satisfies Eq. (11) and the nonflow condition of the liquid through the screen.

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LITERATURE CITED

1. D. N. Gorelov, "Calculation of the aerodynamic interference of a system of bodies in ideal liquids," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekhan. i Mashinostr., No. 5 (1964).